H2 MATHEMATICS

9758/02

Paper 2

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3 hours

Proposed Solutions

Author: Joel Q. L. Chang

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1 We make the following substitutions.

$$\begin{aligned} u &= \sqrt{x+2}, \\ x &= u^2 - 2, \\ \text{“d}x &= 2u \, \mathrm{d}u\text{”}. \quad \color{red}{\text{M1}} \end{aligned}$$

Hence,

$$\begin{aligned} \int \frac{x}{\sqrt{x+2}} \, \mathrm{d}x &= \int \frac{u^2-2}{u} \cdot 2u \, \mathrm{d}u \quad \color{red}{\text{M1}} \\ &= 2\int (u^2-2) \, \mathrm{d}u \\ &= 2\left(\frac{1}{3}u^3 - 2u\right) + C \quad \color{red}{\text{M1}} \\ &= 2\left(\frac{1}{3}(x+2) - 2\right)\sqrt{x+2} + C \\ &= \frac{2}{3}(x-4)\sqrt{x+2} + C. \quad \color{red}{\text{A1}} \end{aligned}$$

2 **(a)** Since the water is poured at a constant rate,

$$\begin{aligned} (\text{volume}) &= (\text{rate}) \times (\text{time}) \\ 0.9h \cdot 1000 &= k \cdot 72 \\ k &= \frac{900h}{72} = 12.5h. \quad \color{red}{\text{A1}} \end{aligned}$$

(b) Let V denote the amount of water at time t. By the question,

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t} &= kt, \\ V &= \int kt \, \mathrm{d}t \\ &= \frac{1}{2}kt^2 + C. \quad \color{red}{\text{M1}} \end{aligned}$$

Since $V(0) = 0$, $C = 0$, and we have

$$V = \frac{1}{2}kt^2.$$

When the container is filled, $V = 0.9h \cdot 1000 = 900h$. Since $t, h > 0$,

$$\begin{aligned} 900h &= \frac{1}{2}kt^2 \quad \color{red}{\text{M1}} \\ &= \frac{1}{2} \cdot 12.5h \cdot t^2 \\ t^2 &= \frac{900}{\frac{1}{2} \cdot 12.5} = 144 \\ t &= 12 \text{ seconds}. \quad \color{red}{\text{A1}} \end{aligned}$$

(c) Let V denote the amount of water at time t. By the question,

$$\frac{\mathrm{d}V}{\mathrm{d}t} = kt + 25,$$
$$V = \frac{1}{2}kt^2 + 25t. \quad \text{M1}$$

Since $V(10) = 900h$,

$$900h = \frac{1}{2} \cdot 12.5h \cdot (10)^2 + 25 \cdot 10 \quad \text{M1}$$
$$h = \frac{250}{275} = \frac{10}{11}\text{ m.} \quad \text{A1}$$

3 **(a)** By direct calculation, M1

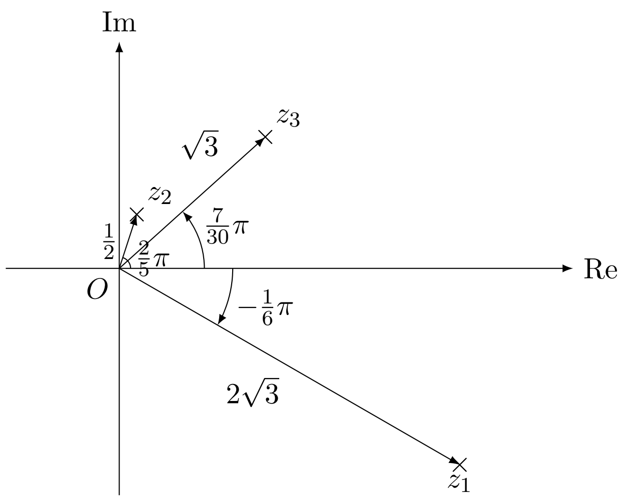
z	$\lvert z\rvert$	$\arg z$	Exponential form of z
z_1	$\sqrt{12}$	$-\frac{1}{6}\pi$	$\sqrt{12}\mathrm{e}^{\mathrm{i}\left(-\frac{1}{6}\pi\right)}$
z_2	$\frac{1}{2}$	$\frac{2}{5}\pi$	$\frac{1}{2}\mathrm{e}^{\mathrm{i}\left(\frac{2}{5}\pi\right)}$

Hence,

$$z_3 = \frac{1}{2}\sqrt{12}\mathrm{e}^{\mathrm{i}\left(-\frac{1}{6}+\frac{2}{5}\right)\pi} = \sqrt{3}\mathrm{e}^{\mathrm{i}\left(\frac{7}{30}\pi\right)}.$$

Therefore, $|z_3| = \sqrt{3}$ A1 and $\arg(z_3) = \frac{7}{30}\pi$. A1

(b) We sketch as follows.



Sketch: M1 M1

(c) Since z_3^n is purely imaginary,

$$\arg(z_3^n) = \underbrace{(2k+1)}_{\text{(odd)}} \cdot \frac{\pi}{2}$$
$$n\arg(z_3) = \left(k + \frac{1}{2}\right)\pi$$
$$n \cdot \frac{7}{30}\pi = \left(k + \frac{1}{2}\right)\pi \quad \text{M1}$$
$$n \cdot \frac{7}{30} = k + \frac{1}{2}$$
$$n = \frac{15}{7}(2k+1).$$

Using G.C., the smallest $n \in \mathbb{Z}^+$ that satisfies this equation is given by $k = 3, n = 15$. A1
Thus,

$$\arg(z_3^n) = \left(3 + \frac{1}{2}\right)\pi = \frac{7}{2}\pi = -\frac{1}{2}\pi, \quad \text{A1}$$
$$|z_3^n| = |z_3|^{15} = (\sqrt{3})^{15} = 2187\sqrt{3}, \quad k = 2187. \quad \text{A1}$$

4 **(a)** Since $9r^2 + 3r - 2 = (3r-1)(3r+2)$, we write

$$\frac{1}{9r^2+3r-2} = \frac{A}{3r-1} + \frac{B}{3r+2},$$
$$1 = A(3r+2) + B(3r-1). \quad \text{M1}$$

Setting $r = \frac{1}{3}$, $A = \frac{1}{3}$. Setting $r = -\frac{2}{3}$, $B = -\frac{1}{3}$. Hence,

$$\frac{1}{9r^2+3r-2} = \frac{1}{3(3r-1)} - \frac{1}{3(3r+2)}. \quad \text{A1}$$

(b) By the method of differences,

$$\begin{aligned}
\sum_{r=m}^{3m} \frac{1}{9r^2+3r-2} &= \sum_{r=m}^{3m} \left(\frac{1}{3(3r-1)} - \frac{1}{3(3r+2)}\right) \\
&= \frac{1}{3(3m-1)} + \frac{1}{3(3m+2)} + \cdots + \frac{1}{3(9m-4)} + \frac{1}{3(9m-1)} \\
&\quad - \frac{1}{3(3m+2)} - \frac{1}{3(3m+5)} - \cdots - \frac{1}{3(9m-1)} - \frac{1}{3(9m+2)} \quad \text{M1} \quad \text{M1} \\
&= \frac{1}{3(3m-1)} - \frac{1}{3(9m+2)} \\
&= \frac{(9m+2)-(3m-1)}{3(3m-1)(9m+2)} \quad \text{M1} \\
&= \frac{2m+1}{(3m-1)(9m+2)}. \quad \text{A1}
\end{aligned}$$

(c) Repeating the previous calculation,

$$\begin{aligned}
\sum_{r=1}^{n} \frac{1}{9r^2+3r-2} &= \sum_{r=1}^{n} \left(\frac{1}{3(3r-1)} - \frac{1}{3(3r+2)}\right) \\
&= \frac{1}{3(2)} - \frac{1}{3(3n+2)} \\
&= \frac{1}{6} - \frac{1}{3(3n+2)}.
\end{aligned}$$

Hence,

$$\sum_{r=1}^{\infty} \frac{1}{9r^2+3r-2} = \lim_{n\to\infty} \left(\sum_{r=1}^{n} \frac{1}{9r^2+3r-2}\right) = \lim_{n\to\infty} \left(\frac{1}{6} - \frac{1}{3(3n+2)}\right) = \frac{1}{6}. \quad \text{B1}$$

(d) We want to find the smallest n such that

$$\left|\sum_{r=1}^{n} \frac{1}{9r^2+3r-2} - \sum_{r=1}^{\infty} \frac{1}{9r^2+3r-2}\right| < 0.004$$
$$\left|\frac{1}{6} - \frac{1}{3(3n+2)} - \frac{1}{6}\right| < 0.004. \quad \text{M1}$$

Using G.C., the smallest value of n is $n = 28$. A1

5 **(a)** Calculating the volume of revolution, the required volume V is,

$$V = \pi \int_{r-h}^{r} x^2 \, \mathrm{d}y = \pi \int_{r-h}^{r} (r^2 - y^2) \, \mathrm{d}y \quad \text{M1}$$
$$= \pi \left[r^2 y - \frac{1}{3} y^3 \right]_{r-h}^{r}. \quad \text{M1}$$

By algebruh,

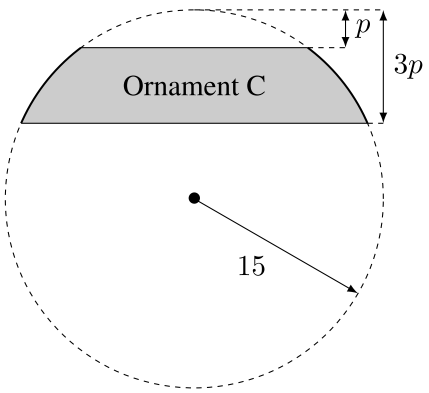
$$\begin{aligned} V = \pi \left[r^2 y - \frac{1}{3} y^3 \right]_{r-h}^{r} &= \pi \left(\left(r^3 - \frac{1}{3} r^3 \right) - \left(r^2 (r-h) - \frac{1}{3} (r-h)^3 \right) \right) \quad \text{M1} \\ &= \pi \left(\frac{2}{3} r^3 - \left((r^3 - r^2 h) - \frac{1}{3} (r^3 - 3r^2 h + 3rh^2 - h^3) \right) \right) \\ &= \frac{1}{3} \pi \left(2r^3 - \left(3(r^3 - r^2 h) - (r^3 - 3r^2 h + 3rh^2 - h^3) \right) \right) \quad \text{M1} \\ &= \frac{1}{3} \pi \left(2r^3 - \left(2r^3 - (3rh^2 - h^3) \right) \right) \\ &= \frac{1}{3} \pi (3rh^2 - h^3) = \frac{1}{3} \pi h^2 (3r - h). \quad \text{M1} \end{aligned}$$

(b) By **(a)**, the volume of the ornament is given by

$$\begin{aligned} 3402\pi &= \frac{4}{3}\pi (15)^3 - \frac{1}{3}\pi p^2 (3 \cdot 15 - p) - \frac{1}{3}\pi (3p)^2 (3 \cdot 15 - 3p) \quad \text{M1} \\ 10\,206 &= 4 \cdot (15)^3 - p^2 (45 - p) - (3p)^2 (45 - 3p) \\ &= 13\,500 - 45p^2 + p^3 - 9p^2 (45 - 3p) \\ &= 13\,500 - 45p^2 + p^3 - 405p^2 + 27p^3 \\ 0 &= 28p^3 - 450p^2 + 3294. \quad \text{A1} \end{aligned}$$

Using G.C., $p = 3, 15.587, -2.5157$.
Since $0 < p < 15$, we reject the latter two options M1 and conclude $p = 3$. A1

(c) By very carefully reading the question, the ornament described in **(c)** looks as follows:



Therefore, the required volume V is

$$\begin{aligned} V &= \frac{1}{3}\pi (3p)^2 (3 \cdot 15 - 3p) - \frac{1}{3}\pi p^2 (3 \cdot 15 - p). \\ &= \frac{1}{3}\pi (9^2 (45 - 9) - 3^2 (42)) \quad \text{M1} \\ &= 846\pi \quad \text{A1} \end{aligned}$$

6 **(a)** Let X denote the throw on which Anil wins. Then

$$\mathbb{P}(\text{Anil wins}) = \mathbb{P}(X=3) + \mathbb{P}(X=5) + \mathbb{P}(X=7) + \dots$$

By observation,

$$\begin{aligned}
\mathbb{P}(X=3) &= \frac{6}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} = \frac{5}{6}\cdot\frac{1}{6},\\
\mathbb{P}(X=5) &= \frac{6}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} = \left(\frac{5}{6}\right)^3\cdot\frac{1}{6},\\
\mathbb{P}(X=7) &= \frac{6}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} = \left(\frac{5}{6}\right)^5\cdot\frac{1}{6}. \quad \text{M1}
\end{aligned}$$

Thus, the required probability is

$$\begin{aligned}
\mathbb{P}(\text{Anil wins}) &= \mathbb{P}(X=3) + \mathbb{P}(X=5) + \mathbb{P}(X=7) + \dots\\
&= \frac{5}{6}\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^3\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^5\cdot\frac{1}{6} + \dots\\
&= \frac{(\text{first term})}{1-(\text{common ratio})} = \frac{\frac{5}{6}\cdot\frac{1}{6}}{1-\left(\frac{5}{6}\right)^2} = \frac{5}{11}. \quad \text{M1} \quad \text{A1}
\end{aligned}$$

(b) By **(a)**, $\mathbb{P}(\text{Babs wins}) = \frac{6}{11}$. M1
Following a similar computation, the required probability is

$$\begin{aligned}
&\mathbb{P}(\text{Babs wins on her 2nd throw} \mid \text{Babs wins})\\
&= \frac{\mathbb{P}(\text{Babs wins on her 2nd throw} \cap \text{Babs wins})}{\mathbb{P}(\text{Bab wins})}\\
&= \frac{\frac{6}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}}{\frac{6}{11}} = \frac{275}{1976}. \quad \text{M1} \quad \text{A1}
\end{aligned}$$

7 **(a)** These 53 employees form a sample, since not every employee replied, and thus the company owner does not have replies from everyone in the department. B1

(b) She could use a random number generator to randomly select $n < 74$ employees. B1
The random number generator allows each person to be chosen with equal probability, thus constituting a random sample. B1

(c) Let a, p, m, s denote the number of Team Leaders in Administration, Production, Marketing, Staffing respectively. We want to find the number of combinations such that

$$\begin{aligned}
a+p+m+s &= 8,\\
a, p, m, s &\geqslant 1,\\
a &> p, m, s.
\end{aligned}$$

We make a few observations:

- If $a = 1, 2$, then there will be at least one department with $\geqslant 2$ Team Leaders, which violates the third condition. Thus, $a \geqslant 3$.
- If $a \geqslant 6$, then at least one of p, m, s is 0, which violates the second condition. Thus, $a \leqslant 5$.

Thus, we check $a = 3, 4, 5$.

(i) If $a = 5$, then automatically p, q, r all equal to 1.
(ii) If $a = 4$, then exactly one of p, q, r is 2, while the other two departments have 1 Team Leader.

(iii) If $a = 3$, then exactly one of p, q, r is 1, while the other two departments have 2 Team Leaders. Thus, the required total is

$$\underbrace{\binom{7}{5}\binom{6}{1}\binom{4}{1}\binom{3}{1}}_{\text{(i)}} + \underbrace{\binom{7}{4}\binom{6}{2}\binom{4}{1}\binom{3}{1} + \binom{7}{4}\binom{6}{1}\binom{4}{2}\binom{3}{1} + \binom{7}{4}\binom{6}{1}\binom{4}{1}\binom{3}{2}}_{\text{(ii)}}$$

$$+ \underbrace{\binom{7}{3}\binom{6}{1}\binom{4}{2}\binom{3}{2} + \binom{7}{3}\binom{6}{2}\binom{4}{1}\binom{3}{2} + \binom{7}{3}\binom{6}{2}\binom{4}{2}\binom{3}{1}}_{\text{(iii)}} \quad \text{M1} \quad \text{M1}$$

$= 1512 + 6300 + 3780 + 2520 + 3780 + 6300 + 9450$ M1 M1

$= 33\,642.$ A1

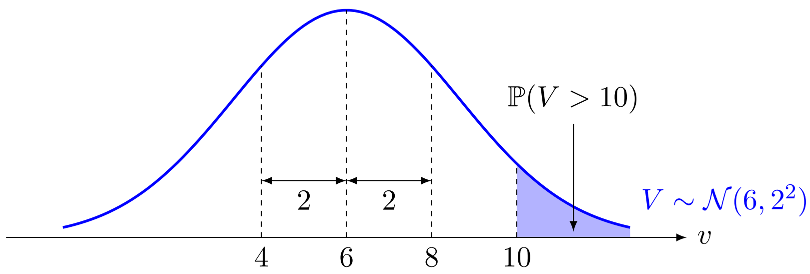
8 **(a)** Since X, Y are independent, $aX - bY$ also follows a normal distribution. We first note that

$$\mathbb{E}[X] = p, \operatorname{Var}(X) = q^2, \mathbb{E}[Y] = s, \operatorname{Var}(Y) = t^2.$$

Thus,

$$\begin{aligned} \mathbb{E}[aX - bY] &= ap - bs, \\ \operatorname{Var}(aX - bY) &= a^2q^2 + b^2t^2, \\ aX - bY &\sim \mathcal{N}(ap - bs, a^2q^2 + b^2t^2). \end{aligned}$$

(b) **(i)** Since $\mathbb{P}(V < 4) = \mathbb{P}(V > 8)$, by symmetry, $\mathbb{E}[V] = \frac{1}{2}(4 + 8) = 6$. B1
Hence, we sketch as follows. B1



(ii) We shade as above. B1
Using G.C., $\mathbb{P}(V > 10) = 0.022750 \approx 0.0228$. B1

(c) By MF26,

$$\begin{aligned} \mathbb{E}[W] &= 1.2\operatorname{Var}(W) \\ 8p &= 1.2 \cdot 8p(1-p) \quad \text{M1} \\ 8p &= 1.2 \cdot 8p(1-p) \\ 1.2 \cdot p(1-p) - p &= 0 \\ p(1 - 6p) &= 0, \end{aligned}$$

thus $p = 0, \frac{1}{6}$. M1 Using G.C.,

$$\begin{aligned} \mathbb{P}(W < 2) &= \mathbb{P}(W \leqslant 1) \\ &= \begin{cases} 1 & \text{if } p = 0, \\ 0.60467 & \text{if } p = \frac{1}{6}, \end{cases} \\ &\approx \begin{cases} 1 & \text{if } p = 0, \\ 0.605 & \text{if } p = \frac{1}{6}. \end{cases} \quad \text{A1} \end{aligned}$$

9 **(a)** Firstly, $\mathbb{P}(\text{L}) = \mathbb{P}(\text{R}) = \frac{1}{2}$.

- Suppose the counter lands at L. Then the probability of landing at B is

$$\frac{4!}{3!1!}p^3q = 4p^3q. \quad \text{M1}$$

- Suppose the counter lands at R. Then the probability of landing at B is p^4.

Therefore, the total probability is

$$\frac{1}{2}(4p^3q) + \frac{1}{2}p^4 = 2p^3q + \frac{1}{2}p^4. \quad \text{M1}$$

(b) Denote the event that Jon and Kathy's counters follow the same route by A and the event that both counters land on B by B. We want to calculate

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}.$$

Firstly, by **(a)**, the probability that Jon and Kath's counters arrive at B is

$$\mathbb{P}(B) = \left(2p^3q + \frac{1}{2}p^4\right)^2.$$

We first observe that if Jon takes a route with probability r, then the probability that both Jon and Kathy take the same route is r^2.

- If Jon's counter lands on L, there are 4 routes, each with probability p^3q.
- If Jon's counter lands on R, there is 1 route, with probability p^4.

Thus, the probability, including the first step, is

$$\mathbb{P}(A \cap B) = 4 \cdot \left(\frac{1}{2}p^3q\right)^2 + 1 \cdot \left(\frac{1}{2}p^4\right)^2 = p^6q^2 + \frac{1}{4}p^8. \quad \text{M1}$$

Thus, the required probability is

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{p^6q^2 + \frac{1}{4}p^8}{\left(2p^3q + \frac{1}{2}p^4\right)^2}. \quad \text{M1}$$

By algebruh,

$$\begin{aligned}
\frac{p^6q^2 + \frac{1}{4}p^8}{\left(2p^3q + \frac{1}{2}p^4\right)^2} &= \frac{\frac{1}{4}p^6(4q^2 + p^2)}{\frac{1}{4}p^6(4q + p)^2} \quad \text{M1} \\
&= \frac{4q^2 + p^2}{(4q + p)^2} \\
&= \frac{4(1-p)^2 + p^2}{(4(1-p) + p)^2} \\
&= \frac{4 - 8p + 5p^2}{(4 - 3p)^2}. \quad \text{A1}
\end{aligned}$$

(c) Following the computation in **(a)**, the probability that the counter lands on C is

$$\frac{1}{2}\cdot\frac{4!}{2!2!}p^2q^2+\frac{1}{2}\cdot\frac{4!}{3!1!}p^3q=3p^2q^2+2p^3q. \quad \text{M1}$$

Since this probability equals that in **(a)**,

$$3p^2q^2+2p^3q=2p^3q+\frac{1}{2}p^4 \quad \text{M1}$$
$$6p^2(1-p)^2=p^4. \quad \text{M1}$$

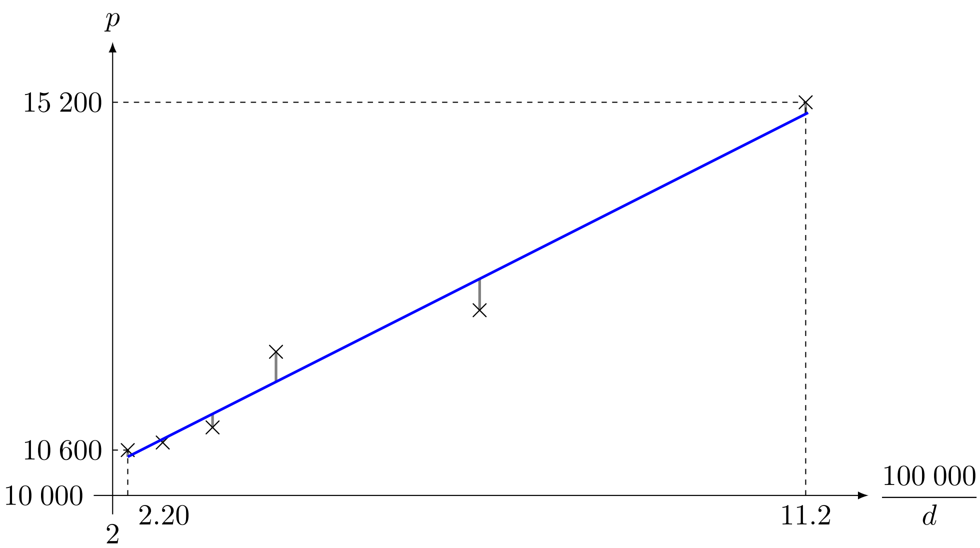
Using G.C., $p=0,0.71010\approx 0,0.710$.
Since the question only requires one value of p, we choose the nontrivial $p=0.710$. A1

10 **(a)** There is a negative correlation between d and p, since p decreases as d increases. B1
The correlation is likely not linear, since on average, p decreases at a gentler rate as d increases. B1

(b) There will be no change to the product moment correlation coefficient, since the correlation does not change due to changes in units. B1

(c) The car is not of the same make nor the same model, since it is a special version, and thus does not represent cars of the same make and the same model in general. Thus, any prediction made thereafter would be unreliable. B1

(d) We sketch the plot in '×' M1 and the line of best fit in blue M1 as follows.



(e) We indicate as above in gray. A1 The distances are squared in order for all errors to be positive. The total error is given by the sum of the squares. B1 Thus, we find the line that minimises the error, i.e., the sum of the squares of these distances, i.e. the 'method of least squares'. B1

(f) Using G.C., the equation of the least squares regression line is

$$p=9398.4925+505.7303\cdot\frac{100\,000}{d}$$
$$p=9400+506\cdot\frac{100\,000}{d}, \text{ (3sf)} \quad \text{A1} \quad \text{A1}$$
$$r=0.98699\approx 0.987. \quad \text{A1}$$

(g) By **(f)**, when $d=5055$,

$$p=9398.4925+505.7303\cdot\frac{100\,000}{5055}=19\,403\approx 19\,400 \text{ pounds sterling.} \quad \text{A1}$$

The estimate would not be reliable since $100\,000/d=19.782\approx 19.7$ lies outside of the data range $[2.20,11.2]$. B1

11 **(a)** Let $X \sim \mathcal{N}(80, 2^2)$ denote Zhou's time and $Y \sim \mathcal{N}(79, 3^2)$ denote Tan's time. We calculate

$$\begin{aligned}\mathbb{E}[X-Y] &= 80-79 = 1,\\ \operatorname{Var}(X-Y) &= 2^2+3^2 = 13,\\ X-Y &\sim \mathcal{N}(1, 13). \quad \text{M1}\end{aligned}$$

Hence, the required probability is

$$\begin{aligned}\mathbb{P}(X<Y) &= \mathbb{P}(X-Y<0) \quad \text{M1}\\ &= 0.39076 \approx 0.391. \quad \text{A1}\end{aligned}$$

(b) The required unbiased estimates $\overline{x}$ and s^2 are given by

$$\begin{aligned}\overline{x} &= \frac{2376.3}{30} = 79.21, \quad \text{A1}\\ s^2 &= \frac{1}{29}\left(188\,653.7 - \frac{2376.3^2}{30}\right) = 14.723 \approx 14.7. \quad \text{A1}\end{aligned}$$

(c) Let μ denote the population mean time, H_0 denote the null hypothesis, and H_1 denote the alternative hypothesis. Our test is as follows.

$$\mathrm{H}_0 : \mu = 80, \quad \mathrm{H}_1 : \mu < 80. \quad \text{M1}$$

Under H_0,

$$\overline{X} \sim \mathcal{N}\left(80, \frac{2^2}{30}\right). \quad \text{M1}$$

The test statistic is

$$Z = \frac{\overline{X}-80}{\sqrt{\frac{2^2}{30}}} \sim \mathcal{N}(0,1).$$

In particular, for $\overline{x} = 79.21$, the test statistic is

$$z = \frac{79.21-80}{\sqrt{\frac{2^2}{30}}} = -2.1635 \approx -2.16. \quad \text{A1}$$

Either of the following two methods: M1

- <u>p-value method</u>: The p-value is given by

$$\mathbb{P}(Z \leqslant -2.1635) = 0.015251 < 0.05.$$

- <u>z-value method</u>: We calculate the critical value

$$z_{\text{crit}} = -1.6448.$$

Since this is a left-tail test, we note that $z = -2.1635 < -1.6448 = z_{\text{crit}}$.

Thus, we reject H_0 and conclude that Zhou's times have reduced. A1

Note: After some thought, I have decided to include the solution involving s^2. I will update this document once there is a finalised consensus on which approach is correct.

Let μ denote the population mean time, H_0 denote the null hypothesis, and H_1 denote the alternative hypothesis. Our test is as follows.

$$\mathrm{H}_0 : \mu = 80, \quad \mathrm{H}_1 : \mu < 80. \quad \text{M1}$$

Under H_0, since $n = 30$ is large, by the central limit theorem,

$$\overline{X} \sim \mathcal{N}\left(80, \frac{14.723}{30}\right) \quad \text{approximately.} \quad \text{M1}$$

The test statistic is

$$Z = \frac{\overline{X} - 80}{\sqrt{\frac{14.723}{30}}} \sim \mathcal{N}(0, 1).$$

In particular, for $\overline{x} = 79.21$, the test statistic is

$$z = \frac{79.21 - 80}{\sqrt{\frac{14.723}{30}}} = -1.1276 \approx -1.13. \quad \text{A1}$$

Either of the following two methods: M1

- p-value method: The p-value is given by

$$\mathbb{P}(Z \leqslant -1.1276) = 0.12972 > 0.05.$$

- z-value method: We calculate the critical value

$$z_{\text{crit}} = -1.6448.$$

  Since this is a left-tail test, we note that $z = -1.1276 > -1.6448 = z_{\text{crit}}$.

Thus, we do not reject H_0 and cannot conclude that Zhou's times have reduced. A1

(d) A 2-tail test as he wants to check whether his times has either increased or decreased. B1

(e) Tan makes the following two assumptions.

1. His times are normally distributed with the same variance. B1
2. Each of his times are independent. B1