H2 MATHEMATICS

Paper 2

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Proposed Solutions

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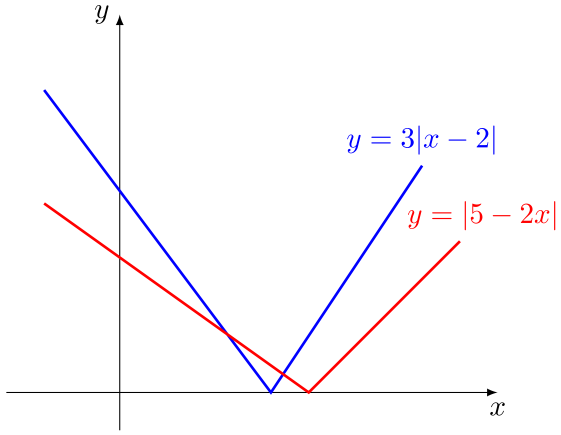
1 **(a)** We sketch as follows.



The graphs intersect at $(1, 3), (2.2, 0.6)$. M1

By the graph, the required set is $\{x \in \mathbb{R} : 1 < x < 2.2\}$. A1

(b) By algebruh,

$$\begin{aligned}\frac{x+25}{x^2-4x-5}+3 &= \frac{x+25+3(x^2-4x-5)}{x^2-4x-5}\\ &= \frac{x+25+3x^2-12x-15}{x^2-4x-5} \qquad \text{M1}\\ &= \frac{3x^2-11x+10}{x^2-4x-5}\\ &= \frac{(3x-5)(x-2)}{(x-5)(x+1)}. \qquad \text{M1}\end{aligned}$$

Hence,

$$\begin{aligned}\frac{x+25}{x^2-4x-5} &> -3\\ \frac{x+25}{x^2-4x-5}+3 &> 0\\ \frac{(3x-5)(x-2)}{(x-5)(x+1)} &> 0\\ (3x-5)(x-2)(x-5)(x+1) &> 0\\ (x+1)(x-5/3)(x-2)(x-5) &> 0. \qquad \text{M1}\end{aligned}$$

Hence, $x < -1$ or $5/3 < x < 2$ or $x > 5$. A1

2 (a) Denote $y' = \frac{\mathrm{d}y}{\mathrm{d}x}, y'' = \frac{\mathrm{d}^2y}{\mathrm{d}x^2}, y^{(3)} = \frac{\mathrm{d}^3y}{\mathrm{d}x^3}, y^{(4)} = \frac{\mathrm{d}^4y}{\mathrm{d}x^4}$ for brevity. By logarithmic properties,

$$y = \ln((\cos(x))^{-1}) = -\ln(\cos(x)).$$

Taking exponentials,

$$e^{-y} = \cos(x).$$

Differentiating with respect to x on both sides,

$$\begin{aligned} e^{-y} \cdot (-y') &= -\sin(x) \\ e^{-y} \cdot y' &= \sin(x). \quad \text{M1} \end{aligned}$$

Differentiating with respect to x on both sides again,

$$\begin{aligned} e^{-y} \cdot (-y') \cdot y' + e^{-y} \cdot y'' &= \cos(x) \\ e^{-y} \cdot (-y') \cdot y' + e^{-y} \cdot y'' &= e^{-y} \\ (-y') \cdot y' + y'' &= 1 \\ y'' &= 1 + (y')^2 \quad \text{M1} \end{aligned}$$

Differentiating with respect to x on both sides again,

$$\begin{aligned} y^{(3)} &= 2 \cdot y' \cdot y'' \\ \frac{\mathrm{d}^3y}{\mathrm{d}x^3} &= 2\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right). \quad \text{M1} \end{aligned}$$

(b) Differentiating with respect to x on both sides again,

$$y^{(4)} = 2 \cdot (y'' \cdot y'' + y' \cdot y^{(3)}). \quad \text{M1}$$

Setting $x = 0$,

$$y = 0, \quad y' = 0, \quad y'' = 1, \quad y^{(3)} = 0, \quad y^{(4)} = 2. \quad \text{M1}$$

Hence, $y \approx \frac{1}{2!}x^2 + \frac{2}{4!}x^4 = \frac{1}{2}x^2 + \frac{1}{12}x^4$. A1

(c) Setting $x = \frac{1}{4}\pi$,

$$\begin{aligned} -\ln\left(\cos\left(\frac{1}{4}\pi\right)\right) &\approx \frac{1}{2}\left(\frac{1}{4}\pi\right)^2 + \frac{1}{12}\left(\frac{1}{4}\pi\right)^4 \\ -\ln\left(\frac{1}{\sqrt{2}}\right) &\approx \frac{1}{2} \cdot \frac{1}{16}\pi^2 + \frac{1}{12} \cdot \frac{1}{256}\pi^4 \quad \text{M1} \\ \frac{1}{2}\ln(2) &\approx \frac{1}{2} \cdot \frac{1}{16}\pi^2 + \frac{1}{2} \cdot \frac{1}{1536}\pi^4 \\ \ln(2) &\approx \frac{1}{16}\pi^2 + \frac{1}{1536}\pi^4. \quad \text{A1} \end{aligned}$$

(d) Using G.C.,

$$\begin{aligned} \int_0^{\frac{1}{10}\pi} \ln(\sec(x))\,\mathrm{d}x &\approx \int_0^{\frac{1}{10}\pi} \left(\frac{1}{2}x^2 + \frac{1}{12}x^4\right)\mathrm{d}x \\ &= \left[\frac{1}{2} \cdot \frac{x^3}{3} + \frac{1}{12} \cdot \frac{x^5}{5}\right]_0^{\frac{1}{10}\pi} \\ &= \frac{1}{6} \cdot \frac{\pi^3}{1000} + \frac{1}{60} \cdot \frac{\pi^5}{100\,000} \\ &= 0.00521871 \approx 0.005219. \quad \text{(4sf)} \quad \text{A1} \end{aligned}$$

3 **(a)** We have

$$\overrightarrow{AB} = \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}, \quad \text{M1}$$

$$\overrightarrow{OC} = \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} + 2\begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ -10 \\ 14 \end{pmatrix}. \quad \text{A1}$$

(b) We have

$$\overrightarrow{AD} = \begin{pmatrix} 1 \\ 2 \\ d \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ d-5 \end{pmatrix}$$

$$\overrightarrow{BD} = \begin{pmatrix} 1 \\ 2 \\ d \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ d-8 \end{pmatrix}$$

Squaring both sides,

$$|\overrightarrow{AD}|^2 = |\overrightarrow{BD}|^2$$

$$2^2 + 0^2 + (d-5)^2 = 0^2 + 4^2 + (d-8)^2 \quad \text{M1}$$

Using G.C., $d = 17/2$. A1

(c) We compute

$$|\overrightarrow{BD}|^2 = |\overrightarrow{AD}|^2 = 2^2 + \left(\frac{17}{2} - 5\right)^2 = \frac{65}{4}.$$

Hence, the required angle $\angle ADB = \theta$ is given by

$$\cos(\theta) = \frac{\overrightarrow{AD} \cdot \overrightarrow{BD}}{|\overrightarrow{AD}||\overrightarrow{BD}|}$$

$$= \frac{\left(\frac{17}{2} - 5\right)\left(\frac{17}{2} - 8\right)}{\frac{65}{4}} = \frac{7}{65} \quad \text{M1} \quad \text{M1}$$

$$\theta = 83.817^\circ \approx 83.8^\circ. \quad \text{A1}$$

(d) Since $|\overrightarrow{AD}| = |\overrightarrow{BD}|$, the diameter of the circle passing D will pass through the midpoint E of AB, computed by

$$\overrightarrow{OE} = \frac{1}{2}\left[\begin{pmatrix}-1\\2\\5\end{pmatrix} + \begin{pmatrix}1\\-2\\8\end{pmatrix}\right] = \begin{pmatrix}0\\0\\13/2\end{pmatrix} \quad \text{M1}$$
$$\overrightarrow{DE} = \begin{pmatrix}0\\0\\13/2\end{pmatrix} - \begin{pmatrix}1\\2\\17/2\end{pmatrix} = \begin{pmatrix}-1\\-2\\-2\end{pmatrix}$$

Hence,

$$l_{DE} : \mathbf{r} = \begin{pmatrix}1\\2\\17/2\end{pmatrix} + \lambda\begin{pmatrix}-1\\-2\\-2\end{pmatrix}, \lambda \in \mathbb{R}. \quad \text{M1}$$

Let F denote the intersection of l_{DE} with the circle. That is,

$$\overrightarrow{OF} = \begin{pmatrix}1\\2\\17/2\end{pmatrix} + \lambda\begin{pmatrix}-1\\-2\\-2\end{pmatrix} \quad \text{for some } \lambda \in \mathbb{R}.$$

The required centre P will then be the midpoint of the diameter DF. Since a triangle in a semicircle is a right-angled triangle, $\angle DBF = 90°$. Hence,

$$\begin{aligned}
0 &= \overrightarrow{BD} \cdot \overrightarrow{BF} \\
&= \begin{pmatrix}0\\4\\1/2\end{pmatrix} \cdot \left(\begin{pmatrix}1\\2\\17/2\end{pmatrix} + \lambda\begin{pmatrix}-1\\-2\\-2\end{pmatrix} - \begin{pmatrix}1\\-2\\8\end{pmatrix}\right) \quad \text{M1} \\
&= \begin{pmatrix}0\\4\\1/2\end{pmatrix} \cdot \left(\begin{pmatrix}0\\4\\1/2\end{pmatrix} + \lambda\begin{pmatrix}-1\\-2\\-2\end{pmatrix}\right) \\
&= \frac{65}{4} - 9\lambda \quad \text{M1} \\
9\lambda &= \frac{65}{4} \\
\lambda &= \frac{65}{36}
\end{aligned}$$

Hence, $\lambda = 65/36$. Therefore, the centre P has position vector

$$\overrightarrow{OP} = \frac{1}{2}\left[\begin{pmatrix}1\\2\\17/2\end{pmatrix} + \begin{pmatrix}1\\2\\17/2\end{pmatrix} + \frac{65}{36}\begin{pmatrix}-1\\-2\\-2\end{pmatrix}\right] = \begin{pmatrix}1\\2\\17/2\end{pmatrix} + \frac{65}{72}\begin{pmatrix}-1\\-2\\-2\end{pmatrix} = \begin{pmatrix}7/72\\7/36\\241/36\end{pmatrix}. \quad \text{A1}$$

4 **(a)** At the x-intercept, $y = 0 \Rightarrow t = 1/5 \Rightarrow x = 77/25$. The required area is

$$\begin{aligned}\int_{\frac{77}{25}}^{21} y \, \mathrm{d}x &= \int_{\frac{1}{5}}^{3} (5t-1)\cdot(4t)\, \mathrm{d}t \quad \text{M1}\\ &= \int_{\frac{1}{5}}^{3} (20t^2 - 4t)\, \mathrm{d}t\\ &= \left[20\cdot\frac{t^3}{3} - 4\cdot\frac{t^2}{2}\right]_{\frac{1}{5}}^{3} \quad \text{M1}\\ &= \left(20\cdot\frac{27}{3} - 4\cdot\frac{9}{2}\right) - \left(20\cdot\frac{1}{375} - 4\cdot\frac{1}{50}\right)\\ &= \frac{12\,152}{75}\ \text{units}^2. \quad \text{A1}\end{aligned}$$

(b) In cartesian form,

$$\begin{aligned} D:&\quad xy = 5u\cdot\frac{4}{u} = 20 \quad\Rightarrow\quad x = \frac{20}{y}\\ C:&\quad x = 2\left(\frac{y+1}{5}\right)^2 + 3 \quad\Rightarrow\quad x = \frac{2}{25}(y+1)^2 + 3. \quad \text{M1}\end{aligned}$$

At their intersection,

$$\begin{aligned}\frac{20}{y} &= \frac{2}{25}(y+1)^2 + 3 \quad \text{M1}\\ 500 &= 2y(y+1)^2 + 75y\\ &= 2y(y^2+2y+1) + 75y\\ &= 2y^3 + 4y^2 + 2y + 75y\\ &= 2y^3 + 4y^2 + 77y\\ 0 &= 2y^3 + 4y^2 + 77y - 500.\end{aligned}$$

Using G.C., $y = 4$ is a real root. M1 By polynomial long division,

$$2y^3 + 4y^2 + 77y - 500 = (2y^2 + 12y + 125)(y-4) = (2(y+3)^2 + 107)(y-4). \quad \text{M1}$$

Thus, $2(y+3)^2 + 107 = 0$ does not yield any real roots. Hence,

$$y = 4 \quad\Rightarrow\quad t = 1 \quad\Rightarrow\quad x = 5$$

and $A(5,4)$. A1

(c) Taking derivatives,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 4t, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = 5 \quad\Rightarrow\quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5}{4t}. \quad \text{M1}$$

At A, $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{t=1} = 5/4$. M1 Hence the tangent to C at A has equation

$$y - 4 = \frac{5}{4}(x-5). \quad \text{M1}$$

At their intersection,

$$\frac{20}{x} - 4 = \frac{5}{4}(x-5). \quad \text{M1}$$

Using G.C., $x = -3.2 \Rightarrow y = -6.25$. Thus, the intersection has coordinates $(-3.2, -6.25)$. A1

5 **(a)** **(i)** We observe that $D \subseteq B$ and $D \subseteq C$. Since A, B are mutually exclusive, so are A, D. B1

(ii) By direct computation,

$$\begin{aligned}\mathbb{P}(A) &= \frac{18}{36} = \frac{1}{2}\\ \mathbb{P}(C) &= \frac{12}{36} = \frac{1}{3}\\ \mathbb{P}(A \cap C) &= \frac{6}{36} = \frac{1}{6}\\ \mathbb{P}(A)\mathbb{P}(C) &= \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}. \quad \text{M1}\end{aligned}$$

Since $\mathbb{P}(A \cap C) = \mathbb{P}(A)\mathbb{P}(C)$, the events A, C are independent. Furthermore, since $B = A'$, A', C are independent. Hence, B, C are independent. A1

(b) **(i)** We still have A, B mutually exclusive, as well as A, D. B1

(ii) We re-compute

$$\begin{aligned}\mathbb{P}(A) &= \frac{18}{35}\\ \mathbb{P}(C) &= \frac{11}{35}\\ \mathbb{P}(A \cap C) &= \frac{6}{35}\\ \mathbb{P}(A)\mathbb{P}(C) &= \frac{18}{35} \cdot \frac{11}{35}.\end{aligned}$$

Since $\mathbb{P}(A \cap C) \neq \mathbb{P}(A)\mathbb{P}(C)$, the events A, C are no longer independent. A1

6 **(a)** By the question,

$$\binom{r}{4}\binom{b}{8} = \binom{r}{3}\binom{b}{9}. \quad \text{M1}$$

By algebruh,

$$\binom{n}{k+1} = \frac{n!}{(k+1)!(n-(k+1))!} = \frac{n-k}{k+1} \cdot \frac{n!}{k!(n-k)!} = \frac{n-k}{k+1}\binom{n}{k}.$$

Hence,

$$\begin{aligned}\frac{r-3}{4} \cdot \binom{r}{3}\binom{b}{8} &= \binom{r}{3} \cdot \frac{b-8}{9} \cdot \binom{b}{8}\\ 9(r-3) &= 4(b-8)\\ 9r - 27 &= 4b - 32\\ 9r + 5 &= 4b. \quad \text{M1}\end{aligned}$$

(b) Similar to **(a)**,

$$\begin{aligned}\binom{r}{3}\binom{b}{9} &= \frac{5}{3}\binom{r}{2}\binom{b}{10} \quad \text{M1}\\ \frac{r-2}{3} \cdot \binom{r}{2}\binom{b}{9} &= \frac{5}{3}\binom{r}{2} \cdot \frac{b-9}{10}\binom{b}{9}\\ \frac{r-2}{3} &= \frac{5}{3} \cdot \frac{b-9}{10} \quad \text{M1}\\ 2(r-2) &= b-9\\ 2r-4 &= b-9\\ 2r+5 &= b. \quad \text{A1}\end{aligned}$$

Thus, we want to solve

$$\begin{aligned} 9r - 4b &= -5, \\ 2r - b &= -5. \quad \text{M1} \end{aligned}$$

Using G.C., $(r, b) = (15, 35)$. M1 Thus, the required probability is

$$\frac{\binom{15}{1}\binom{35}{11}}{\binom{50}{12}} = 0.051551 \approx 0.0516. \quad \text{A1}$$

7 **(a)** **(i)** Using G.C., $r_{\textbf{(i)}} \approx 0.9281$. (4dp) A1

(ii) Using G.C., $r_{\textbf{(ii)}} \approx 0.9697$. (4dp) A1

(b) The model $e^y = cx + d \Leftrightarrow y = \ln(cx + d)$ gives a better fit A1 since $|r_{\textbf{(ii)}}| \approx 0.9697$ is closer to 1 than $|r_{\textbf{(i)}}| \approx 0.9281$. M1 Using G.C., $c = 375.615$, $d = -1881.482$. Hence, the model is given by

$$e^y = 376x - 1880. \quad \text{(3sf)} \quad \text{A1}$$

(c) Using G.C., $c = 375.615$, $d = -1881.482$. Setting $x = 24$,

$$y \approx \ln(375.615 \cdot 24 - 1881.482) = 8.8725 \approx 8.87. \quad \text{A1}$$

There are approximately 8.87 million mobile phone subscriptions in 2024. This estimate is not reliable since $24 \notin [4, 18]$ lies outside the data range, and the estimation is an extrapolation. B1

8 **(a)** **(i)** Let μ denote the population average mass of granulated sugar per bag, in kg, that the new machine fills. B1 The hypotheses are given by

$$\mathrm{H}_0 : \mu = 1, \quad \mathrm{H}_1 : \mu \neq 1. \quad \text{B1}$$

(ii)

1. The machine may not be operating in similar conditions in the morning compared to the afternoon, yielding a possibly non-independent sample. B1
2. Since the distribution of the amount of granulated sugar produced is unknown, a sample of size $10 < 30$ is too small for $\overline{X}$ to be estimated by a normal distribution. B1

(b) **(i)** Let μ denote the population average mass of granulated sugar per bag, in kg, that the new machine fills. The hypotheses are given by

$$\mathrm{H}_0 : \mu = 2, \quad \mathrm{H}_1 : \mu < 2. \quad \text{A1}$$

We compute the unbiased estimate for the population variance (s^2) as follows:

$$s^2 = \frac{1}{39}\left(155.6746 - \frac{78.88^2}{40}\right) = \frac{79}{25\,000} \quad \text{M1}$$

Assume H_0 holds. Since the sample size $n = 40$ is large, by the central limit theorem,

$$\overline{X} \sim \mathcal{N}\left(\mu, \frac{s^2}{n}\right) \quad \text{approximately.} \quad \text{M1}$$

Since the hypothesis test is left-tailed, the critical region is given by $(-\infty, c)$, where

$$\mathbb{P}(\overline{X} \leqslant c) = 0.025.$$

By standardisation,

$$\begin{aligned}\mathbb{P}\left(Z \leqslant \frac{c-\mu}{s/\sqrt{n}}\right) &= 0.025 \\ \frac{c-\mu}{s/\sqrt{n}} &= -1.95996 \quad \text{M1} \\ c &= \mu - 1.95996 \cdot \frac{s}{\sqrt{n}} \\ &= 2 - 1.95996 \cdot \frac{\sqrt{79/25000}}{\sqrt{40}} \\ &= 1.9825 \approx 1.98.\end{aligned}$$

Thus, the critical region is $\overline{x} \in (-\infty, 1.98)$. A1

(c) We compute the sample mean by

$$\overline{x} = \frac{74.88}{40} = 1.972 \in (-\infty, 1.98). \quad \text{A1}$$

Thus, there is sufficient evidence to reject H_0, i.e., the average mass of sugar in the bags are less than 2 kg. A1

9 **(a)** Let $L \sim \mathcal{N}(1.82, 0.2^2)$ denote the length, in metres, of a Long plank. Using G.C.,

$$\mathbb{P}(L < 1.79) = 0.44038 \approx 0.440. \quad \text{A1}$$

(b) Denote $\overline{L} = \frac{1}{8}(L_1 + \cdots + L_8) \sim \mathcal{N}\big(1.82, 0.2^2/8\big)$. Using G.C.,

$$\mathbb{P}(L_1 + \cdots + L_8 > 14.5) = \mathbb{P}\left(\overline{L} > \frac{14.5}{8}\right) = 0.54223 \approx 0.542.$$

(c) Let $R \sim \mathcal{N}(1.22, 0.3^2)$ denote the length, in metres, of a Regular plank. We first compute

$$\mathbb{P}(R > 1.25) = 0.46017. \quad \text{M1}$$

Let $M \sim \mathrm{B}(120, 0.46017)$ denote the number of Regular planks, out of 120, that are longer than 1.25 m. By MF26,

$$\mathbb{E}[M] = 120 \cdot 0.46017 = 120 \cdot 55.2204 \approx 55.2 \text{ m}. \quad \text{M1} \quad \text{A1}$$

(d) Denote $W := (L_1 + \cdots + L_{10}) - (R_1 + \cdots + R_{16})$. Then

$$\begin{aligned}\mathbb{E}[W] &= 10 \cdot 1.82 - 16 \cdot 1.22 = -1.32, \quad \text{M1} \\ \mathrm{Var}(W) &= 10 \cdot 0.2^2 + 16 \cdot 0.3^2 = 1.84.\end{aligned}$$

Hence, $W \sim \mathcal{N}(-1.32, 1.84)$. M1
It follows that

$$\mathbb{P}(|W| < 0.65) = \mathbb{P}(-0.65 < W < 0.65) = 0.23746 \approx 0.237. \quad \text{A1}$$

(e) Denote $V := \frac{1}{3}L - \frac{1}{2}R \sim \mathcal{N}(-0.0033333, 0.026944)$. M1 M1 Then

$$\mathbb{P}\left(\frac{1}{3}L > \frac{1}{2}R\right) = \mathbb{P}(V > 0) = 0.49189 \approx 0.492. \quad \text{M1} \quad \text{A1}$$

(f) Assuming the same material and cross-sectional area, the variance of V would increase, leading to a slightly higher probability. B1

10 **(a)** **(i)** 1. The probability that a glass ornament is faulty remains constant. B1

2. The event that a glass ornament is faulty is independent of the event that other glass ornaments are faulty. B1

(ii) Let $X \sim \mathrm{B}(50, 0.04)$ denote the number of ornaments, out of 50, that are faulty. By MF26,

$$|\mathbb{E}[X] - \mathrm{Var}(X)| = |50 \cdot 0.04 - 50 \cdot 0.04 \cdot 0.96| = 0.08. \quad \text{M1}$$

(iii) Using G.C., $\mathbb{P}(X \leqslant 2) = 0.67671 \approx 0.677$. A1

(iv) Let $Y \sim \mathrm{B}(5, 0.67671)$ denote the number of days, out of 5, where no more than 2 faulty ornaments are produced. M1 Using G.C.,

$$\mathbb{P}(Y \geqslant 3) = 1 - \mathbb{P}(Y \leqslant 2) = 0.80477 \approx 0.805. \quad \text{M1} \quad \text{A1}$$

(v) Let $W \sim \mathrm{B}(250, 0.04)$ denote the number of items, out of 250, that are produced. M1 Using G.C.,

$$\mathbb{P}(Y \leqslant 10) = 0.58305 \approx 0.583. \quad \text{A1}$$

(b) The probability of a pen being faulty is $1 - p$. Let p_L denote the probability that Mr Lu accepts a box of pens, and p_M denote the probability that Mrs Ming accepts a box of pens. Let $V \sim \mathrm{B}(6, 1-p)$ denote the number of faulty pens out of 6. Then

$$\begin{aligned} p_\mathrm{L} &= \mathbb{P}(V \leqslant 1) \\ &= \mathbb{P}(V = 0) + \mathbb{P}(V = 1) \\ &= p^6 + 6(1-p)p^5. \quad \text{M1} \end{aligned}$$

Let $U \sim \mathrm{B}(3, 1-p)$ denote the number of faulty pens out of 3. Then

$$\begin{aligned} p_\mathrm{M} &= \mathbb{P}(U = 0) + \mathbb{P}(U = 1) \cdot \mathbb{P}(U = 0) \\ &= p^3 + 3(1-p)p^2 \cdot p^3 \quad \text{M1} \\ &= p^3 + 3(1-p)p^5. \quad \text{M1} \end{aligned}$$

By algebruh,

$$\begin{aligned} p_\mathrm{M} - p_\mathrm{L} &= p^3 + 3(1-p)p^5 - (p^6 + 6(1-p)p^5) \quad \text{M1} \\ &= p^3 - p^6 - 3(1-p)p^5 \\ &= p^3(1 - p^3 - 3(1-p)p^2) \\ &= p^3(1 - 3p^2 + 2p^3) \\ &= p^3(p-1)(2p^2 - p - 1) \quad \text{M1} \\ &= p^3(p-1)(2p+1)(p-1) \\ &= p^3(p-1)^2(2p+1) \\ &= p^3(1-p)^2(2p+1) > 0. \quad \text{M1} \end{aligned}$$

Thus, $p_\mathrm{M} > p_\mathrm{L}$, as required.