

2025 GCE A-Level H2 Mathematics Paper 2

Examination Solutions

Joel Kindiak

Questions have not been uploaded due to copyright reasons. Do comment if you spot any errors.

Section A: Pure Mathematics [40 marks]

Question 1. System of Linear Equations

Solution. (a) Using the question,

$$\begin{cases} 8c + 11t + 5b = 114, \\ 5c + 14t + 7b = 112, \\ 9c + 9t + 4b = 110. \end{cases}$$

Solving the system of linear equations, $c = 7, t = 3, b = 5$.

(b) Since Sam came second and every player scored a different number of points, Sam scored fewer than Pritti's 114 points but more than Quentin's 112 points, i.e. 113 points. Let x, y, z denote the number of car, train, and boat tokens respectively that Sam has. Hence,

$$7x + 3y + 5z = 113.$$

Furthermore, we are given that $x \geq 6, y \geq 6, z \geq 6$, so that

$$7x + 3y + 5z \geq 90.$$

By fixing $y = z = 6$ for simplicity, we must have $7x \leq 65 \Rightarrow x \leq 9$. Similarly, $y \leq 13$ and $z \leq 10$.

By **trial-and-error**, $x = 6, y = 7, z = 10$ works, with x, y, z distinct and $z > y$. Since there is only one solution, Sam had 6 car tokens, 7 train tokens, and 10 boat tokens.

Question 2. Vectors

Solution. (a) Firstly,

$$\overrightarrow{PQ} = \begin{bmatrix} 1 \\ 6 \\ 6 \end{bmatrix} - \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}.$$

Since $\overrightarrow{QR} = \frac{4}{3}\overrightarrow{PQ}$ and P, Q, R are collinear, we have

$$\begin{aligned}\overrightarrow{PR} &= \overrightarrow{PQ} + \overrightarrow{QR} \\ &= \overrightarrow{PQ} + \frac{4}{3}\overrightarrow{PQ} \\ &= \frac{7}{3}\overrightarrow{PQ} = \frac{7}{3} \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ 14 \end{bmatrix}.\end{aligned}$$

Hence,

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR} = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 7 \\ 7 \\ 14 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 14 \end{bmatrix}.$$

(b) Firstly,

$$\overrightarrow{SQ} = \begin{bmatrix} 1 \\ 6 \\ 6 \end{bmatrix} - \begin{bmatrix} -1 \\ -2 \\ c \end{bmatrix}.$$

Since $\overrightarrow{SQ} \perp \overrightarrow{PQ}$:

$$\begin{aligned}\left(\begin{bmatrix} 1 \\ 6 \\ 6 \end{bmatrix} - \begin{bmatrix} -1 \\ -2 \\ c \end{bmatrix} \right) \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} &= 0 \\ (1 + 6 + 12) - (-1 - 2 + 2c) &= 0 \\ 19 - (2c - 3) &= 0 \\ c &= 11.\end{aligned}$$

(c) Hence,

$$\overrightarrow{PS} = \begin{bmatrix} -1 \\ -2 \\ 11 \end{bmatrix} - \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 11 \end{bmatrix}.$$

Therefore the required angle θ is given by

$$\begin{aligned}\begin{bmatrix} 1 \\ -5 \\ 11 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} &= \left\| \begin{bmatrix} 1 \\ -5 \\ 11 \end{bmatrix} \right\| \left\| \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\| \cos \theta \\ 18 &= \sqrt{147} \sqrt{6} \cos \theta \\ \cos \theta &= \frac{18}{\sqrt{147} \sqrt{6}} \\ \theta &\approx 52.7^\circ.\end{aligned}$$

Question 3. Differential Equations

Solution. (a) By the method of separable variables,

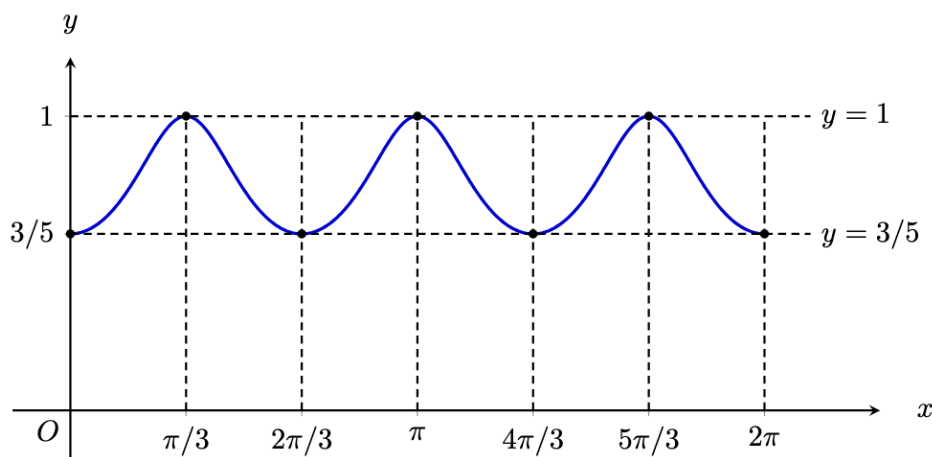
$$\begin{aligned}\int y^{-2} dy &= \int \sin 3x dx \\ \frac{y^{-1}}{-1} &= -\frac{1}{3} \cos 3x + C \\ \frac{1}{y} &= \frac{1}{3} \cos 3x - C \\ y &= \frac{1}{-C + \frac{1}{3} \cos 3x}.\end{aligned}$$

Since the initial condition is $y(\pi) = 1$,

$$1 = \frac{1}{-C + \frac{1}{3} \cos 3\pi} \Rightarrow C = -\frac{4}{3}.$$

Therefore, $y = \frac{1}{\frac{4}{3} + \frac{1}{3} \cos 3x} = \frac{3}{4 + \cos 3x}$, where $A = 3, B = 4$.

(b) (i) By the graph $c = 3/5, 1$ (these values correspond with $\cos 3x = \pm 1$).



(b) (ii) We first note that the graph of $y = f(x)$ is defined on all of $\mathbb{R}$, not just the domain $[0, 2\pi]$. By considering reflections about vertical lines, we observe that the lines of symmetry of $y = f(x)$ are all of the form $x = k\pi/3$, where k is any integer (positive, zero, or negative). On the sketched domain $0 \leq x \leq 2\pi$, these are $x = k\pi/3$ for $k = 1, \dots, 5$.

(b) (iii) Since $\cos$ is 2π -periodic,

$$\begin{aligned}\cos(3(x + \pi)) &= \cos(3x + 3\pi) \\ &= \cos(3x + \pi) \\ &= -\cos 3x.\end{aligned}$$

Hence,

$$f(x + \pi) = \frac{A}{B + \cos(3(x + \pi))} = \frac{A}{B - \cos 3x} = \frac{3}{4 - \cos 3x}.$$

(c) Using the product rule and the chain rule,

$$\begin{aligned}
 \frac{d^2y}{dx^2} &= \frac{d}{dx}(y^2 \sin 3x) \\
 &= 2y \cdot \frac{dy}{dx} \cdot \sin 3x + y^2 \cdot 3 \cos 3x \\
 &= 2y \cdot y^2 \sin 3x \cdot \sin 3x + y^2 \cdot 3 \cos 3x \\
 &= (3 \cos 3x) \cdot y^2 + (2 \sin^2 3x) \cdot y^3,
 \end{aligned}$$

where $P(x) = 3 \cos 3x$, $Q(x) = 2 \sin^2 3x$.

Question 4. Rate of Change

Solution. (a) Letting T denote the time taken to fill the bowl,

$$10\pi \cdot T = \frac{1}{3}\pi \cdot 15^2 \cdot (45 - 15) \Rightarrow T = 225 \text{ s.}$$

(b) Using the chain rule,

$$\begin{aligned}
 \frac{dV}{dt} &= \frac{dV}{dh} \cdot \frac{dh}{dt} \\
 &= \frac{1}{3}\pi \cdot (2h \cdot (45 - h) + h^2 \cdot (-1)) \cdot \frac{dh}{dt} \\
 &= \frac{1}{3}\pi \cdot h(90 - 3h) \cdot \frac{dh}{dt}.
 \end{aligned}$$

At $h = 12$,

$$10\pi = \frac{1}{3}\pi \cdot 12(90 - 3 \cdot 12) \cdot \frac{dh}{dt} \Big|_{h=12} \Rightarrow \frac{dh}{dt} \Big|_{h=12} = \frac{5}{108} \text{ cm s}^{-1}.$$

(c) The total volume poured into the bowl after T seconds is

$$\int_0^T 10\pi t \, dt = 10\pi \cdot \frac{T^2}{2} = 5\pi \cdot T^2.$$

Since $T > 0$, the required time taken is given by solving the equation:

$$5\pi \cdot T^2 = 972\pi \Rightarrow T = \sqrt{194.4} \approx 13.9 \text{ s.}$$

(d) At $V = 972\pi$,

$$\frac{1}{3}\pi h^2(45 - h) = 972\pi \Rightarrow h^3 - 45h^2 + 2916 = 0.$$

Solving the cubic equation, $h = 43.455, 9, -7.4558$. Since $h \in [0, 15]$, we reject the first and last option and conclude $h = 9$. By (c), this occurs at time $\tau = \sqrt{194.4}$ s, when water is being poured at the rate $10\pi\tau \text{ cm}^3 \text{ s}^{-1}$.

Therefore,

$$10\pi\tau = \frac{1}{3}\pi \cdot 9(90 - 3 \cdot 9) \cdot \frac{dh}{dt} \Big|_{h=9} \Rightarrow \frac{dh}{dt} \Big|_{h=9} = \frac{10\tau}{189} \approx 0.738 \text{ cm s}^{-1}.$$

Section B: Probability and Statistics [60 marks]

Question 5. Probability

Solution. (a) The total number of counters is $3 + 5 + 7 = 15$. The probability that the counters are of the same colours is given by

$$\frac{3 \times 2 + 5 \times 4 + 7 \times 6}{15 \times 14} = \frac{34}{105}$$

Hence, the required probability is its complement $1 - \frac{34}{105} = \frac{71}{105}$.

(b) The first red counter is the n -th counter taken if the first $n - 1$ counters are all non-red (there are 12 non-red counters) and the n -th counter is red. Hence, the required equation to solve is:

$$\frac{12!}{(13 - n)!} \cdot \frac{(16 - n)!}{15!} \cdot \frac{3}{16 - n} = \frac{36}{455}.$$

The left-hand side, using factorials, simplifies to

$$\frac{12!}{(13 - n)!} \cdot \frac{(16 - n)!}{15!} \cdot \frac{3}{16 - n} = \frac{(15 - n)(14 - n)}{910}.$$

Hence, we need to solve the quadratic equation

$$\frac{(15 - n)(14 - n)}{910} = \frac{36}{455}.$$

Solving, $(15 - n)(14 - n) = 72$ yields $n = 6$ or $n = 23$. Since at most 12 non-red counters can precede the first red counter, $n \leq 13$, so that $n = 6$.

Question 6. Normal Distribution

Solution. (a) Using the symmetry of the normal distribution,

$$\mathbb{P}(X < 7) = \mathbb{P}(X < \mu) - \mathbb{P}(7 \leq X < \mu) = 0.5 - 0.45 = 0.05.$$

(b) Since $X \sim \mathcal{N}(\mu, 1.8^2)$, we have

$$0.05 = \mathbb{P}(X < 7) = \mathbb{P}\left(Z < \frac{7 - \mu}{1.8}\right).$$

Using z -scores,

$$\frac{7 - \mu}{1.8} = -1.6449 \quad \Rightarrow \quad \mu = 7 + 1.6449 \cdot 1.8 = 9.96082 \approx 9.96.$$

(c) Using independence,

$$\begin{aligned} 0.38 &= \mathbb{P}((X > 7) \cap (Y > 7)) = \mathbb{P}(X > 7) \cdot \mathbb{P}(Y > 7) \\ &= 0.95 \cdot \mathbb{P}(Y > 7). \end{aligned}$$

By algebra,

$$\mathbb{P}(Y > 7) = \frac{0.38}{0.95} = 0.4 \quad \Rightarrow \quad \mathbb{P}(Y \leq 7) = 0.6.$$

Since $Y \sim \mathcal{N}(\lambda, 2.8^2)$,

$$\mathbb{P}\left(Z \leq \frac{7 - \lambda}{2.8}\right) = \mathbb{P}(Y \leq 7) = 0.6.$$

Using z -scores,

$$\frac{7 - \lambda}{2.8} = 0.25334 \quad \Rightarrow \quad \lambda = 7 - 2.8 \cdot 0.25334 = 6.2906 \approx 6.29.$$

(d) Using complements,

$$\begin{aligned}\mathbb{P}(Y \leq a) &= 1 - \mathbb{P}(Y > a) \\ &= 1 - 2 \cdot \mathbb{P}(Y > 7) \\ &= 1 - 2 \cdot 0.4 = 0.2.\end{aligned}$$

Since $Y \sim \mathcal{N}(6.2906, 2.8^2)$ by (c), $a \approx 3.93$.

Question 7. Permutations and Combinations

Solution. (a) The total number of arrangements, including repetitions, is

$$\frac{8!}{2! \cdot 2!} = 10\,080.$$

(b) We first choose the start and end numbers, then place the third even number in the middle four spaces, and then permute the odd numbers:

$$(3 \times 2) \times \binom{4}{1} \times \frac{5!}{2! \cdot 2!} = 720.$$

(c) The total number of even-number groups (consisting of 2, 4, 8) is $3! = 6$. Hence, the required total is

$$3! \cdot \frac{6!}{2! \cdot 2!} = 1080.$$

(d) We first count the number of arrangements where the even digits are all together and the odd digits are all together:

$$3! \cdot \frac{5!}{2! \cdot 2!} \cdot 2! = 360.$$

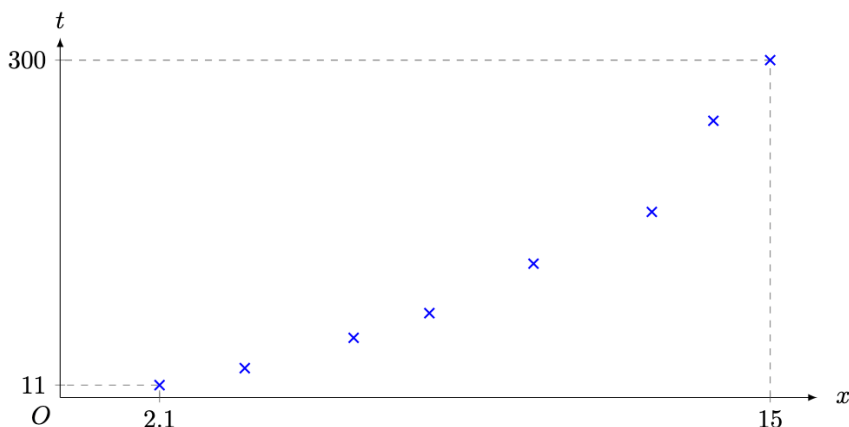
The required probability is given by a conditional probability as follows:

$$\frac{360}{1080} = \frac{1}{3}.$$

Question 8. Correlation and Linear Regression

Solution. (a) Since $r = 0.39 \in (0.3, 0.7)$, there is a weak positive linear correlation between the best performances of the athletes in the high jump and long jump events. This is consistent with the scatter diagram, where the points do not lie close to a straight line.

(b) (i) The diagram suggests that there is a positive, though non-linear, correlation between x and t , where t increases at an increasing rate as x increases.



(ii) (A) For the model $t = ax + b$, we obtain a correlation coefficient of $r_1 \approx 0.958$, and for the model $t = cx^2 + d$ we obtain a correlation coefficient of $r_2 \approx 0.990$. Since $0 < r_1 < r_2 < 1$ (i.e. $|r_2|$ is closer to 1 than $|r_1|$), the model $t = cx^2 + d$ is a better fit to the data.

(ii) (B) The regression equation is

$$t = 1.24969x^2 + 0.87580$$

$$t = 1.25x^2 + 0.876. \quad (3\text{sf})$$

(iii) (A) At $x = 11$,

$$t = 1.24969 \cdot 11^2 + 0.87580 \approx 152 \text{ minutes.}$$

(iii) (B) Firstly, since $r_2 \approx 0.990$ is close to 1, the linear correlation between t and x^2 is strongly positive. Secondly, $x = 11 \in (2.1, 15)$ lies within the range of the data, so the estimate is an interpolation. Hence, the estimate is reliable.

Question 9. Binomial Distribution

Solution. (a) The two assumptions are as follows:

- The probability that a randomly chosen refrigerator is faulty is constant.
- The faultiness of any refrigerator is independent of other refrigerators.

(b) Since $X \sim B(90, 0.02)$, the required probability is

$$\mathbb{P}(X > 1) = 1 - \mathbb{P}(X \leq 1) = 1 - 0.46043 = 0.53956 \approx 0.540.$$

(c) Let $V \sim B(5, 0.53956)$ denote the number of days in a 5-day working week, on which Alan finds more than one faulty refrigerator. The required probability is

$$\mathbb{P}(V < 2) = \mathbb{P}(V \leq 1) = 0.14195 \approx 0.142.$$

(d) Since $90 \times 5 = 450$, let $W \sim B(450, 0.02)$ denote the number of faulty refrigerators that Alan finds in total, in a 5-day work week. Then the required probability is

$$\mathbb{P}(W < 10) = \mathbb{P}(W \leq 9) = 0.58741 \approx 0.587.$$

(e) Since $X \sim B(90, 0.02)$ and $Y \sim B(60, 0.03)$ are independent, the required probability is given by three cases:

$$\begin{aligned} \mathbb{P}(X + Y = 2) &= \mathbb{P}(X = 2) \cdot \mathbb{P}(Y = 0) + \mathbb{P}(X = 1) \cdot \mathbb{P}(Y = 1) + \mathbb{P}(X = 0) \cdot \mathbb{P}(Y = 2) \\ &= 0.27074 \cdot 0.16080 + 0.29812 \cdot 0.29840 + 0.16231 \cdot 0.27225 \\ &= 0.17668 \approx 0.177. \end{aligned}$$

Question 10. Flour-Production

Solution. (a) In the question, X denotes the mass, in kg, of packets produced by Machine P , and Y denotes the mass, in kg, of packets produced by Machine Q . Since the machines are different and not related in general, the masses of packets of flour produced would also not affect one another. Therefore, X, Y are independent.

(b) Let $X \sim \mathcal{N}(2.2, 0.1^2)$ denote the mass of a randomly chosen packet of flour produced by Machine P . Let $Y \sim \mathcal{N}(2.1, 0.05^2)$ denote the mass of a randomly chosen packet of flour produced by Machine Q . Then

$$\begin{aligned} \mathbb{E}[X - Y] &= 2.2 - 2.1 = 0.1, \\ \text{Var}(X - Y) &= 0.1^2 + 0.05^2 = 0.0125, \\ X - Y &\sim \mathcal{N}(0.1, 0.0125). \end{aligned}$$

Hence, the required probability is

$$\mathbb{P}(X > Y) = \mathbb{P}(X - Y > 0) = 0.81445 \approx 0.814.$$

(c) Defining the random variable $W := X_1 + X_2 + X_3 + Y_1 + \cdots + Y_5$,

$$\begin{aligned} \mathbb{E}[W] &= 3 \cdot 2.2 + 5 \cdot 2.1 = 17.1, \\ \text{Var}(W) &= 3 \cdot 0.1^2 + 5 \cdot 0.05^2 = 0.0425. \end{aligned}$$

Hence, $W \sim \mathcal{N}(17.1, 0.0425)$, so that the required probability is

$$\mathbb{P}(W > 17) = 0.68616 \approx 0.686.$$

(d) Let μ denote the mean mass of a packet of flour, in kg, produced by the adjusted Machine P . Then the null hypothesis H_0 and the alternative hypothesis H_1 are given as follows:

$$H_0 : \mu = 2.2, \quad H_1 : \mu \neq 2.2.$$

(e) Let $\bar{x}$ and s^2 denote the unbiased estimates of the population mean and variance of the relevant masses. Then

$$\bar{x} = 2 + \frac{4.5}{30} = 2.15$$

and

$$s^2 = \frac{1}{30 - 1} \left(1.11 - \frac{4.5^2}{30} \right) = 0.015.$$

(f) Suppose H_0 holds. Since the sample size $n = 30$ is sufficiently large, by the central limit theorem,

$$\bar{X} \sim \mathcal{N}\left(2.2, \frac{0.015}{30}\right) \quad \text{approximately.}$$

The test statistic is $z = \frac{2.15 - 2.2}{\sqrt{0.015/30}} \approx -2.24$. Since we are conducting a two-tailed test, the required p -value is given by

$$p = 2 \cdot \mathbb{P}(\bar{X} \leq 2.15) = 2 \cdot 0.012673 \approx 0.0253 < 0.05.$$

Since $p < 0.05$, there is sufficient evidence to reject H_0 and conclude that the mean mass of packets produced by the adjusted Machine P differs from 2.2 kg.

(g) The distribution of the masses after the adjustment is unknown. With a sample of fewer than 30 packets, the sample may be too small for the central limit theorem to apply, so $\bar{X}$ may not be approximately normal and the p -value calculated in (f) may be inaccurate.

(h) A sample that is not chosen randomly may not be representative of all packets produced by the adjusted Machine P , so the sample mean may be biased. Furthermore, the masses in such a sample may not be independent of one another, so that $\bar{X}$ may not follow a normal distribution, even approximately.

—Joel Kindiak, 7 Nov 25, 1522H