2026 GCE A-Level H2 Mathematics Paper 2
Examination Solutions

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Section A: Pure Mathematics [40 marks]

Question 1. Vectors

Solution. A vector perpendicular to both given vectors is their cross product:

$$\begin{aligned}\begin{bmatrix}3\\-2\\10\end{bmatrix}\times\begin{bmatrix}2\\3\\-2\end{bmatrix}&=\begin{bmatrix}(-2)(-2)-(10)(3)\\(10)(2)-(3)(-2)\\(3)(3)-(-2)(2)\end{bmatrix}\\&=\begin{bmatrix}-26\\26\\13\end{bmatrix}=13\begin{bmatrix}-2\\2\\1\end{bmatrix}.\end{aligned}$$

Since $\sqrt{(-2)^2+2^2+1^2}=3$, a unit vector perpendicular to both given vectors is

$$\frac{1}{3}\begin{bmatrix}-2\\2\\1\end{bmatrix}=-\frac{2}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}.$$

Remark 1. The negative of this vector is the only other unit vector perpendicular to both given vectors.

Question 2. Volumes of Revolution

Solution. **(a) (i)** The volume generated is

$$\begin{aligned}V&=\pi\int_0^h(kx)^2\,\mathrm{d}x\\&=\pi k^2\left[\frac{x^3}{3}\right]_0^h=\frac{1}{3}\pi k^2h^3.\end{aligned}$$

(ii) Rotating the line $y=(r/h)x$, for $0\le x\le h$, through 2π radians about the x-axis generates a cone of height h, whose base has radius $y=r$ at $x=h$. Setting $k=r/h$ in **(i)**,

$$V=\frac{1}{3}\pi\left(\frac{r}{h}\right)^2h^3=\frac{1}{3}\pi r^2h.$$

(b) Substituting $y = kx$ and $x = 3/5$ into the equation of the curve, and then multiplying by $25/3$,

$$\begin{aligned}\left(\frac{3}{5} - k\right)^2 + \left(\frac{3}{5}k\right)^2 &= k^2 \\ \frac{9}{25} - \frac{6}{5}k + \frac{9}{25}k^2 &= 0 \\ 3k^2 - 10k + 3 &= 0 \\ (3k-1)(k-3) &= 0.\end{aligned}$$

Thus $k = 1/3$ or $k = 3$. Since $y = kx = 3k/5$ at this point, the possible values of y are $1/5$ and $9/5$.

Question 3. Complex Numbers

Solution. **(a)** Since the coefficients p and q are real, the conjugate $1 - 2i$ is also a root, so that

$$\begin{aligned}(z - (1+2i))(z - (1-2i)) &= (z-1)^2 - (2i)^2 \\ &= z^2 - 2z + 5\end{aligned}$$

is a factor of the cubic. Let γ be the third root. Then, for every z,

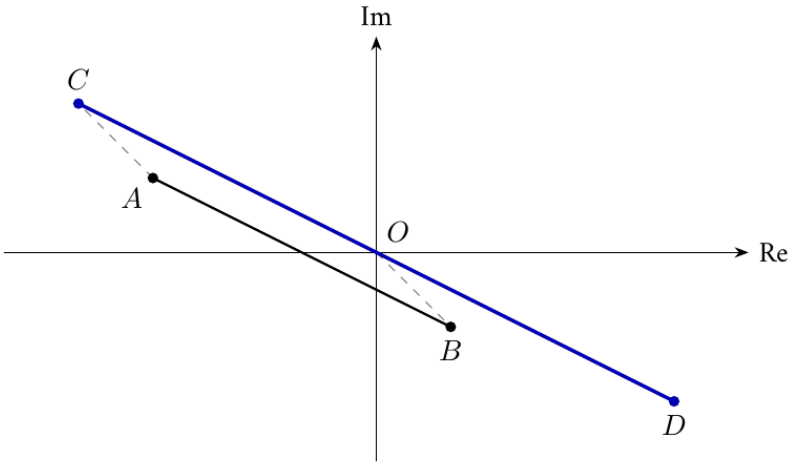
$$\begin{aligned}z^3 + pz^2 - z + q &= (z^2 - 2z + 5)(z - \gamma) \\ &= z^3 - (2+\gamma)z^2 + (5 + 2\gamma)z - 5\gamma.\end{aligned}$$

Comparing the coefficients of z, $5 + 2\gamma = -1$, so that $\gamma = -3$. Then comparing the coefficients of z^2 and the constant terms,

$$p = -(2+\gamma) = 1, \quad q = -5\gamma = 15.$$

Therefore, the other roots are $1 - 2i$ and -3, with $p = 1$ and $q = 15$.

(b) (i) Since $w_3 = w_1 - w_2$ represents the vector $\overrightarrow{BA}$, the point C satisfies $\overrightarrow{OC} = \overrightarrow{BA}$, so that $OBAC$ is a parallelogram. Furthermore, $w_4 = -(w_1 - w_2) = -w_3$, so D is the image of C under a rotation of π radians about O.



Therefore, C and D are symmetric about the origin: O is the midpoint of CD, with $OC = OD$.

(ii) Taking the conjugate of a complex number reflects its point in the real axis. Since E and F represent w_1^* and w_2^*, the line AB is mapped onto the line EF by a reflection in the real axis.

Question 4. Arithmetic and Geometric Series

Solution. **(a)** Equating the given terms,

$$a + 5d = ar^4,$$
$$a + 21d = ar^6.$$

The first equation yields $5d = a(r^4 - 1)$. Multiplying the second equation by 5, substituting, and then dividing by $a \neq 0$,

$$5a + 21a(r^4 - 1) = 5ar^6$$
$$5r^6 - 21r^4 + 16 = 0$$
$$(r^2 - 1)(5r^2 + 4)(r^2 - 4) = 0.$$

Since $5r^2 + 4 > 0$, we have $r^2 = 1$ or $r^2 = 4$. If $r^2 = 1$, then $r^4 = 1$ and $d = 0$, which is impossible. Hence $r^2 = 4$, and since $r > 0$,

$$r = 2, \quad d = \frac{a(2^4 - 1)}{5} = 3a.$$

Conversely, for every non-zero real a, the values $r = 2$ and $d = 3a \neq 0$ satisfy all the given conditions. Therefore, the set of possible values of a is $\{a \in \mathbb{R} : a \neq 0\}$.

(b) Since $d = 3a$ and $r = 2$, the two sums are

$$S_{37} = \frac{37}{2}\left(2a + 36(3a)\right) = 2035a,$$
$$T_n = \frac{a(2^n - 1)}{2 - 1} = a(2^n - 1).$$

Since the sums differ by $12a$, and dividing by $a \neq 0$,

$$|2035 - (2^n - 1)| = 12 \quad \iff \quad |2036 - 2^n| = 12$$
$$\iff \quad 2^n = 2024 \quad \text{or} \quad 2^n = 2048.$$

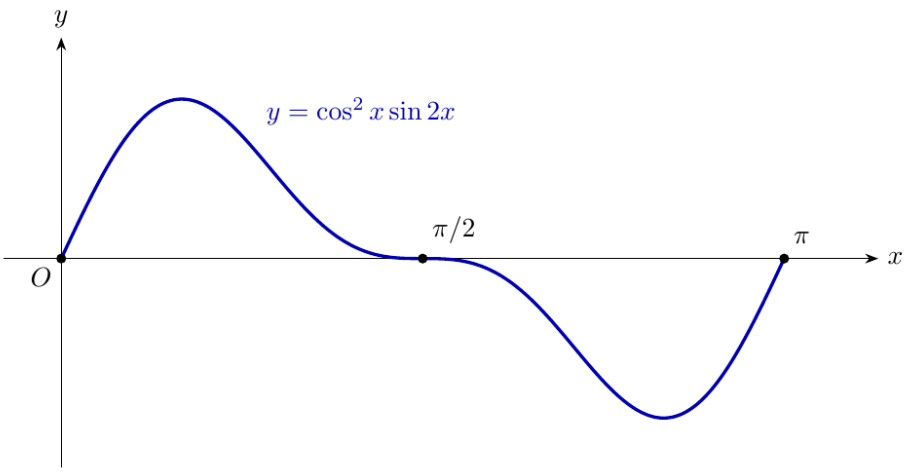
Since $2^{10} = 1024$ and $2^{11} = 2048$, the number 2024 is not a power of 2. Therefore, $n = 11$.

Question 5. Applications of Differentiation and Integration

Solution. **(a)** By the double angle formula $\sin 2x = 2\sin x \cos x$,

$$y = \cos^2 x \sin 2x = 2\sin x \cos^3 x.$$

For $0 \leq x \leq \pi$, we have $y = 0$ exactly when $\sin x = 0$ or $\cos x = 0$, that is, $x = 0$, $\pi/2$ or π. Thus the curve meets the x-axis at $(0, 0)$, $(\pi/2, 0)$ and $(\pi, 0)$. Furthermore, $y > 0$ for $0 < x < \pi/2$ and $y < 0$ for $\pi/2 < x < \pi$. Because of the factor $\cos^3 x$, the curve crosses the x-axis at $(\pi/2, 0)$ with zero gradient.



(b) By the product rule and the chain rule,

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= 2\cos x \cos^3 x + 2\sin x \cdot 3\cos^2 x(-\sin x) \\ &= 2\cos^2 x\left(\cos^2 x - 3\sin^2 x\right).\end{aligned}$$

Stationary points therefore occur where $\cos x = 0$ or $\tan^2 x = 1/3$, that is, $x = \pi/2$, $\pi/6$ or $5\pi/6$. The maximum point lies where $y > 0$, so $x = \pi/6$, and

$$y = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3\sqrt{3}}{8}.$$

Therefore, the maximum point is $(\pi/6, 3\sqrt{3}/8)$.

(c) Since $y \geq 0$ for $0 \leq x \leq \pi/4$, and $\dfrac{\mathrm{d}}{\mathrm{d}x}(\cos^4 x) = -4\cos^3 x \sin x$, the required area is

$$\begin{aligned}\int_0^{\pi/4} 2\sin x \cos^3 x \,\mathrm{d}x &= \left[-\frac{1}{2}\cos^4 x\right]_0^{\pi/4} \\ &= -\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^4 + \frac{1}{2} \\ &= \frac{1}{2} - \frac{1}{8} = \frac{3}{8}.\end{aligned}$$

Section B: Probability and Statistics [60 marks]

Question 6. Sampling, Permutations and Combinations

Solution. **(a)** The manager wishes to learn the opinions of all the employees of the company, so the population consists of all $22 + 18 + 9 + 6 = 55$ employees. The 40 employees who return the questionnaire are only part of this population, so they comprise a sample.

Remark 2. This sample is not representative of the population, since no employees from the Green team or the Yellow team are included.

(b) In a random sample, every employee has the same chance of being chosen, so the committee is free from any bias in the manager's choice of employees, and is more likely to represent the opinions of all the employees.

(c) Let r, b, g and y be the numbers of members from the Red, Blue, Green and Yellow teams, with $r+b+g+y = 5$, $g \geq 1$, and r greater than each of b, g and y.

If $r \geq 4$, then the other teams have at most 1 member altogether, so at most two teams are represented. If $r \leq 1$, then $b = g = y = 0$, which is impossible. If $r = 2$, then b, g and y are each at most 1, with sum 3. If $r = 3$, then $b + g + y = 2$, with $g \geq 1$ and a third team represented. Therefore, the possible cases are as follows:

$$\begin{aligned}
(r,b,g,y) = (2,1,1,1): &\quad \binom{22}{2}\binom{18}{1}\binom{9}{1}\binom{6}{1} = 224\,532,\\
(r,b,g,y) = (3,1,1,0): &\quad \binom{22}{3}\binom{18}{1}\binom{9}{1} = 249\,480,\\
(r,b,g,y) = (3,0,1,1): &\quad \binom{22}{3}\binom{9}{1}\binom{6}{1} = 83\,160.
\end{aligned}$$

Therefore, the committee can be formed in $224\,532 + 249\,480 + 83\,160 = 557\,172$ ways.

Question 7. Probability

Solution. **(a)** Let there be r red, y yellow and b blue balls, where $r + y + b = 20$. There are $\binom{20}{2} = 190$ equally likely pairs of balls. A pair with one red ball and one yellow ball is formed by choosing 1 of the r red balls and 1 of the y yellow balls, in ry ways. Similarly, rb pairs consist of one red ball and one blue ball. Therefore,

$$\begin{aligned}
\frac{ry}{190} = \frac{28}{95} \quad &\Rightarrow \quad ry = 56,\\
\frac{rb}{190} = \frac{4}{19} \quad &\Rightarrow \quad rb = 40.
\end{aligned}$$

Adding the equations, and then using $y + b = 20 - r$,

$$\begin{aligned}
r(20 - r) &= 96\\
r^2 - 20r + 96 &= 0\\
(r - 8)(r - 12) &= 0.
\end{aligned}$$

If $r = 12$, then $y = 56/12$ is not an integer. Therefore, there were 8 red balls originally in the bag, with 7 yellow balls and 5 blue balls.

(b) (i) The events $A \cap B$, $A \cap B'$, $A' \cap B$ and $(A \cup B)'$ partition the sample space, so that

$$\mathbb{P}(A \cap B) = 1 - 0.364 - 0.124 - 0.276 = 0.236 \neq 0.$$

Therefore, A and B are not mutually exclusive.

(ii) Using **(i)**,

$$\mathbb{P}(A) = \mathbb{P}(A \cap B') + \mathbb{P}(A \cap B) = 0.364 + 0.236 = 0.6,$$
$$\mathbb{P}(B) = \mathbb{P}(A' \cap B) + \mathbb{P}(A \cap B) = 0.124 + 0.236 = 0.36.$$

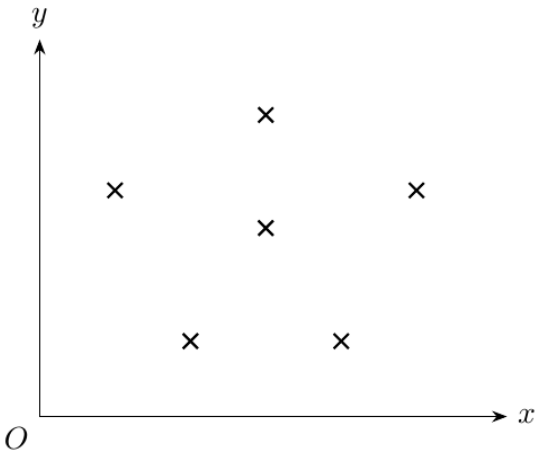
Then $\mathbb{P}(A) \cdot \mathbb{P}(B) = 0.216 \neq 0.236 = \mathbb{P}(A \cap B)$. Therefore, A and B are not independent.

Question 8. Correlation and Linear Regression

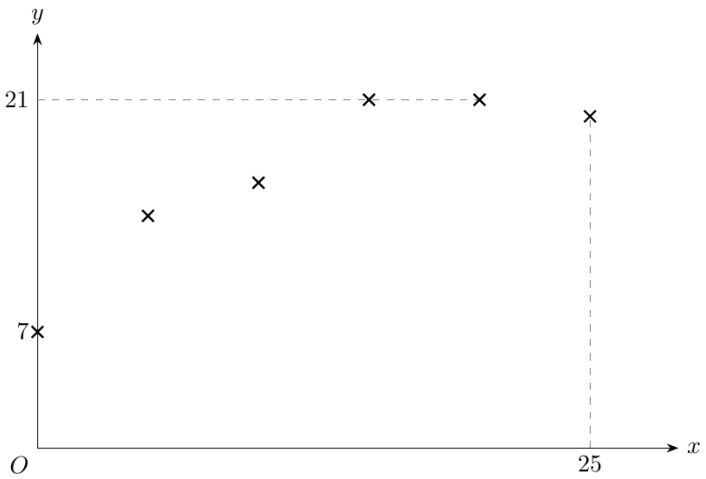
Solution. **(a)** Tang should draw a scatter diagram of the data to check whether the points lie close to a straight line. A product moment correlation coefficient close to 1 does not by itself imply that the relationship is linear, since points lying on a curve can also give such a value.

(b) (i) Since the product moment correlation coefficient is approximately 0, there is no linear correlation between the amount of fertiliser used and the crop yield. However, Xi should not conclude that the crop yield is unrelated to the amount of Fertiliser B used, since there may still be a non-linear relationship between them.

(ii) The following scatter diagram shows no linear trend, and its product moment correlation coefficient is 0.

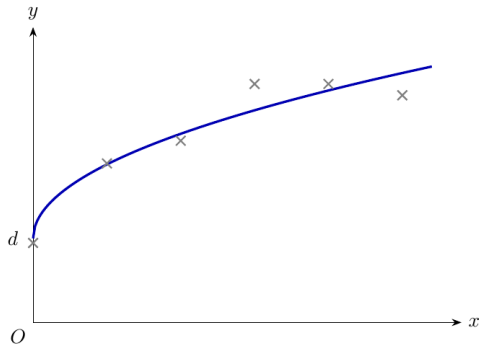


(c) (i) The scatter diagram of Leia's data is as follows.

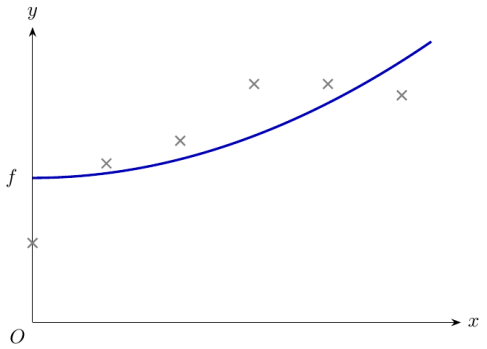


As x increases, y increases at a decreasing rate, and levels off from $x = 15$ onwards, with a slight decrease at $x = 25$. The points lie on a curve rather than close to a straight line, so the crop yield is not linearly related to the amount of fertiliser used.

(ii) The sketches of the two models, with the data points shown in grey, are as follows.



$y = c\sqrt{x} + d$



$y = ex^2 + f$

Since c and e are positive, the curve $y = c\sqrt{x} + d$ increases at a decreasing rate, whereas $y = ex^2 + f$ increases at an increasing rate. The data show y increasing at a decreasing rate, so $y = c\sqrt{x} + d$ is the better model.

Remark 3. This agrees with the product moment correlation coefficients: 0.966 between $\sqrt{x}$ and y, but only 0.734 between x^2 and y.

(iii) Using the GC, the least squares regression line of y on $\sqrt{x}$ is

$$y = 2.91126\sqrt{x} + 7.40549.$$

Therefore, when $x = 13$, the estimated crop yield is

$$y = 2.91126\sqrt{13} + 7.40549 = 17.90218 \approx 17.9 \text{ kg per square metre.}$$

Remark 4. This estimate is reliable, since $x = 13$ lies within the range of the data and the product moment correlation coefficient between $\sqrt{x}$ and y is close to 1.

Question 9. Binomial Distribution

Solution. **(a)** The 100 packets contain 1200 Tringos altogether, of which the number of green Tringos is

$$0(15) + 1(32) + 2(27) + 3(18) + 4(6) + 5(2) = 174.$$

Therefore, the estimated probability that a randomly chosen Tringo is green is $174/1200 = 0.145$.

(b) Let $R \sim \mathrm{B}(12, 0.3)$ denote the number of red sweets in a packet. The required probability is

$$\mathbb{P}(R < 5) = \mathbb{P}(R \leq 4) = 0.72366 \approx 0.724.$$

(c) Let $V \sim \mathrm{B}(7, 0.72366)$ denote the number of packets, out of 7, that contain fewer than 5 red sweets. The required probability is

$$\mathbb{P}(V \geq 4) = 1 - \mathbb{P}(V \leq 3) = 0.90255 \approx 0.903.$$

(d) Since a carton contains $72 \times 12 = 864$ randomly chosen Tringos, let $T \sim \mathrm{B}(864, 0.3)$ denote the number of red sweets in a carton. The required probability is

$$\mathbb{P}(T > 250) = 1 - \mathbb{P}(T \leq 250) = 0.73987 \approx 0.740.$$

(e) Let $Y \sim \mathrm{B}(12, p)$ denote the number of yellow sweets in a packet. Then

$$\begin{aligned}\mathbb{P}(Y = 2) + \mathbb{P}(Y = 3) &= \binom{12}{2} p^2 (1-p)^{10} + \binom{12}{3} p^3 (1-p)^9 \\ &= 22p^2(1-p)^9(3 + 7p).\end{aligned}$$

Solving $22p^2(1-p)^9(3+7p) = 0.206$ for $0 < p < 1$ using the GC, $p \approx 0.072$ or $p \approx 0.400$.

Question 10. Normal Distribution

Solution. **(a) (i)** Let $X \sim \mathcal{N}(22, 3^2)$ denote the time taken, in minutes, for a journey between Aytown and Beetown using the usual route. The required probability is

$$\mathbb{P}(22.5 < X < 24) = 0.18132 \approx 0.181.$$

(ii) Let $S \sim \mathcal{N}(75, 12^2)$ denote the time spent at the shops, and let X_1 and X_2 denote the times for the two journeys. Defining $W := X_1 + S + X_2$,

$$\begin{aligned} \mathbb{E}[W] &= 22 + 75 + 22 = 119, \\ \mathrm{Var}(W) &= 3^2 + 12^2 + 3^2 = 162. \end{aligned}$$

Hence $W \sim \mathcal{N}(119, 162)$, and since 2 hours is 120 minutes, the required probability is

$$\mathbb{P}(W < 120) = 0.53131 \approx 0.531.$$

(iii) Let $Y \sim \mathcal{N}(25, 4^2)$ denote the time taken using the alternative route. Then

$$\begin{aligned} \mathbb{E}[Y - X] &= 25 - 22 = 3, \\ \mathrm{Var}(Y - X) &= 4^2 + 3^2 = 25. \end{aligned}$$

Hence $Y - X \sim \mathcal{N}(3, 5^2)$, and the required probability is

$$\mathbb{P}(Y < X) = \mathbb{P}(Y - X < 0) = 0.27425 \approx 0.274.$$

(b) Let $U \sim \mathcal{N}(\mu, \sigma^2)$ denote the time taken, in minutes, for a journey from Aytown to Seatown. Then $\mathbb{P}(U < 55) = 0.873$ and $\mathbb{P}(U \leq 49) = 1 - 0.471 = 0.529$. Using z-scores,

$$\begin{aligned} \frac{55 - \mu}{\sigma} &= 1.14069, \\ \frac{49 - \mu}{\sigma} &= 0.072756. \end{aligned}$$

Subtracting the second equation from the first, $6/\sigma = 1.06793$, so that

$$\sigma = 5.6183, \quad \mu = 55 - 1.14069 \times 5.6183 = 48.591.$$

Therefore, the required probability is

$$\mathbb{P}(45 < U < 50) = 0.33765 \approx 0.338.$$

Question 11. Hypothesis Testing

Solution. **(a) (i)** Let μ grams denote the population mean mass of bird seed in a packet. The hypotheses are

$$\mathrm{H}_0 : \mu = 400, \quad \mathrm{H}_1 : \mu \neq 400.$$

(ii) The unbiased estimates of the population mean and variance are

$$\bar{x} = 400 + \frac{120}{40} = 403,$$
$$s^2 = \frac{1}{39}\left(1902 - \frac{120^2}{40}\right) = \frac{1542}{39} = 39.538.$$

Since $n = 40$ is large, by the Central Limit Theorem, $\overline{X} \sim \mathcal{N}(400, 39.538/40)$ approximately under H_0. For a two-tailed test at the 5% level, H_0 is rejected when

$$\left|\frac{\bar{x} - 400}{\sqrt{39.538/40}}\right| \geq 1.95996 \quad \Longleftrightarrow \quad |\bar{x} - 400| \geq 1.9486$$
$$\Longleftrightarrow \quad \bar{x} \leq 398.05 \quad \text{or} \quad \bar{x} \geq 401.95.$$

Therefore, correct to 1 decimal place, the critical region is $\bar{x} \leq 398.1$ or $\bar{x} \geq 401.9$.

(iii) Since $\bar{x} = 403 \geq 401.9$ lies in the critical region, we reject H_0. There is sufficient evidence at the 5% level of significance to conclude that the mean mass of bird seed in a packet is not 400 grams.

(b) (i) Let μ grams denote the population mean mass of pet food in a bag. The owner is testing whether the mean mass of pet food in a bag filled by the machine is greater than the stated 500 grams.

(ii) For a one-tailed test at the 5% level, the critical z-value is 1.645, correct to 4 significant figures.

(iii) Firstly, the sample is not random. Only the last 5 bags filled on Monday, Wednesday and Friday can be chosen, so bags filled earlier in the day, or on Tuesday or Thursday, have no chance of being selected. Bags filled at the end of the day may also differ systematically from the others.

Secondly, the sample of 15 bags is small, so the Central Limit Theorem cannot be used to approximate the distribution of the sample mean by a normal distribution. The z-test is therefore valid only if the masses are known to be normally distributed, which is not given.

Remark 5. The diagrams have been drawn and the solutions have been verified with the help of Generative AI.

—Joel Kindiak, 6 Nov 26, 1410H